Substitution in algebra

Remember...
In an algebraic expression, letters are used as substitutes for numbers.

Example 1
Find the value of the following expressions if $s = 82$.

a) $s - 35 + 18$
$82 - 35 + 18$
$= 100 - 35 = 65$

b) $4s + 15 - 60$
$4 \times 82 + 15 - 60$
$= 328 - 45 = 283$

Example 2
Find the value of the following expressions if $a = 3$ and $b = 8$.

a) $5(a + b)$
$5 \times (3 + 8)$
$= 5 \times 11 = 55$

b) $2(b + 3a) - (4a + b)$
$2 \times (8 + 3 \times 3) - (4 \times 3 + 8)$
$= 2(8 + 9) - (12 + 8)$
$= 2 \times 17 - 20 = 14$

1 Find the value of the following expressions if $x = 39$.

a) $43 + x - 24 =$

b) $4x - 45 + 27 =$

c) $67 - \frac{x}{3} - 17 =$

d) $47 - 2x + 59 + 4x =$

e) $84 + 5x - 92 - 57 =$

f) $42 - 2x + 138 - 3x + 58 =$

2 Find the value of the following expressions if $c = 5$ and $d = 8$.

a) $3c - d + 24 =$

b) $34 + 5(d - c) =$

c) $4d - 84 + 8c + 68 =$

d) $12(d + c) - 7(d - c) =$

e) $\frac{40}{d} - \frac{15}{c} =$

f) $6(3c - 2d) + 6d - 6c =$

3 Find the value of the following expressions if $u = 4$, $k = 7$ and $t = 9$.

a) $12(t - 6) + 4(u + k) =$

b) $7k - 4(t - k) + 6u =$

c) $51 - 3(k + u) + 5(t - u) =$

d) $8(2u + k) - 3(t - k) =$

e) $29 - 2(3t - 4u) + 5(k + t) =$

f) $4k - 7(3u - t) - 6(2u - k) =$

4 Find the value of the following expressions if $x = 3$, $y = 6$ and $z = 12$.

a) $47 + \dfrac{3z}{3x} =$

b) $\dfrac{48}{2y} + 3z =$

c) $3(4y - z) - \dfrac{8x}{z} =$

d) $8z - \dfrac{24x}{2z} + 4y =$

e) $43(2z - 3y) - 21(4y - 7x) + \dfrac{51}{x} =$

f) $7z + 2(9y - x) - 4(5y + x) =$

5 The letters c and d stand for two whole numbers.

a) Which two numbers could c and d stand for if $cd = 24$?

b) What if $cd = 24$ and $c - d = 5$?
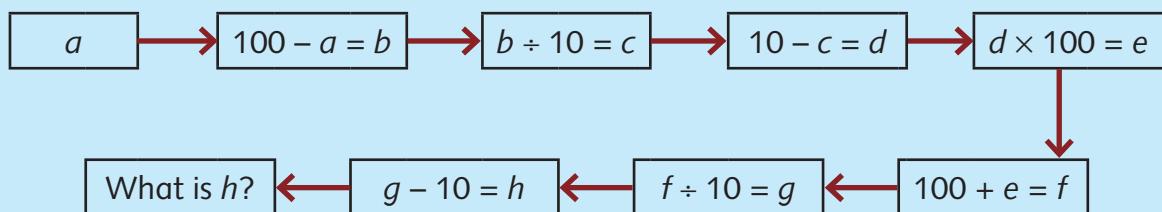Look at your answers to part a) to find c and d.

c) $cd = 30$ and $c - d = 7$
What is the value of c and d?

6 The letters x and y stand for two whole numbers.

a) $x = y - 20$ and $x + y = 60$
What is the value of x and y?

b) $x = 2y$ and $x + y = 60$
What is the value of x and y?

Try this

a → $100 - a = b$ → $b \div 10 = c$ → $10 - c = d$ → $d \times 100 = e$

What is h? ← $g - 10 = h$ ← $f \div 10 = g$ ← $100 + e = f$

a) Choose a 2-digit number. This number is a.

b) Feed it into the flowchart. What number is h?

c) Try this with other 2-digit numbers. What do you notice? Why?

d) What would happen if a was a 1-digit number?

Simplifying algebraic expressions

Letters can be used to stand for an unknown number.
As with arithmetic, in algebra, do the division first then the multiplication, then addition, then the subtraction.

Example 1	Example 2	Example 3
Simplify $12a - 4a + a + 2a - 6a$	Simplify $18s \div 3$	Simplify $\dfrac{9n}{8c} \times \dfrac{24c}{3}$
$12a - 4a + a + 2a - 6a$ $= 12a + a + 2a - 4a - 6a$ $= 15a - 10a$ $= 5a$	$18s \div 3$ $= \dfrac{18s}{3}$ $= \dfrac{18 \times s}{3}$ $= 6 \times s$ $= 6s$	$\dfrac{^3\cancel{9} \times n}{\cancel{8} \times \cancel{c}} \times \dfrac{^3\cancel{24} \times \cancel{c}}{\cancel{3}}$ $= 3n \times 3$ $= 9n$

1 Simplify each of the following expressions.

a) $3z - 8z + 7z + 19z$

b) $2p + 5p - 4p - 6p + 7p$

c) $v - 3v + 8v - 3v - 2v$

d) $4c - 6b - 2c + 12b$

e) $5f - 4u - 3f + 17f - 6u$

f) $x + 4x - 2w + 3x + 7w$

g) $7t - 4r - 2r + 6r - 3t$

h) $7d + 9g - 3g + 4d - 2g$

i) $12k + 4d + 2d - 5d - 6k$

j) $3e + 5e - 7h - 12h - 4e$

k) $4u + 2s - 7s - 3s + 8u$

l) $8y - 4a + 12y - 3y + 9a$

m) $3c - 5v - 8c + 6g + 2v$

n) $12h - 4s + 3h - 6t + 9s$

o) $2q + 4q - 5t - 7z + 2z$

p) $4g - 8c - 2g - 5c + 6d$

q) $5s - 12t - 14t + 6h - 11s$

r) $4j - 4r + 5j - 4d - 9d$

s) $b + 5h + 3h - 17b - 6h + 7g$

t) $2u - 4u + 7j + 4j - 6f - 5u$

u) $5r + 3r + 8t - 6y - 3y - 12r$

2 Each of the following algebraic expressions has been simplified. Write two different expressions involving addition and subtraction for each before they were simplified.

a) $7y$

b) $3a$

c) $12t$

d) $2x + 3y$

e) $4e - 8d$

3 Simplify each of the following expressions.

a) $7 \times 6m$

b) $4a \times 12$

c) $14w \div 7$

d) $24t \div 8$

e) $8c \times \frac{3}{4}$

f) $12k \times \frac{13}{4}$

g) $8b \times 7c$

h) $3z \times 21y$

i) $24h \times \frac{j}{6}$

j) $\frac{2u}{3} \times 15k$

k) $16t \times \frac{23g}{8}$

l) $\frac{14f}{15} \times \frac{9d}{2}$

m) $\frac{4n}{7} \times \frac{49r}{2}$

n) $\frac{51s}{9} \times \frac{36g}{17}$

o) $\frac{14q}{15} \times \frac{3}{7s}$

p) $\frac{9}{16h} \times \frac{10r}{15}$

q) $\frac{6d}{7e} \times \frac{21e}{9}$

r) $\frac{4}{7u} \times \frac{11u}{16f}$

s) $\frac{8b}{11p} \times \frac{13}{24b}$

t) $\frac{5z}{9n} \times \frac{36n}{25z}$

u) $\frac{17j}{8y} \times \frac{64y}{51j}$

4 Each of the following algebraic expressions has been simplified. Write two different expressions involving multiplication and division for each before they were simplified.

a) $5g$

b) $2c$

c) $8d$

d) $10t$

e) $3a$

Try this

Choose 12 of the expressions you simplified in questions 1 and 3 and check your answers by substituting numbers for letters.

Example
Try the expression and your answer.
If $z = 2$

1 **a)**

$3z - 8z + 7z + 19z \longrightarrow 21z$

$6 - 16 + 14 + 38 \longrightarrow 21 \times 2 = 42$

$= 58 - 16 = 42$

Equations

Equations have symbols or letters to represent unknown numbers. When you find the value of the letter you have solved the equation.

Example 1	**Example 2**	**Example 3**
$62 + g = 147$ $g = 147 - 62$ So $g = 85$ To check: $62 + 85 = 147$	$3a + 4 = 19$ $3a = 19 - 4 = 15$ So $a = 15 \div 3$ $a = 5$ To check: $3 \times 5 + 4 = 19$	$\dfrac{a}{9} = 18$ $\dfrac{a}{9} \times 9 = 18 \times 9$ (To find out what a stands for we multiply both sides by 9.) $a = 162$ To check: $\dfrac{a}{9} = \dfrac{162}{9} = 18$

1 Write the value of each letter.

a) $c - 34 = 52$ $c = \boxed{}$ **b)** $27 + f = 61$ $f = \boxed{}$

c) $36 - s = 19$ $s = \boxed{}$ **d)** $y + 18 = 54$ $y = \boxed{}$

e) $2t = 46$ $t = \boxed{}$ **f)** $4m = 84$ $m = \boxed{}$

g) $k - 13 = 47$ $k = \boxed{}$ **h)** $28 + j = 43$ $j = \boxed{}$

i) $3n = 42$ $n = \boxed{}$ **j)** $g - 35 = 17$ $g = \boxed{}$

2 Work out the value of each letter. Show all your working.

a) $9z = 24.3$ **b)** $\dfrac{t}{7} = 84$ **c)** $11.5 = 23s$

d) $\dfrac{g}{12} = 47$ **e)** $14.5 = \dfrac{3k}{18}$ **f)** $371 = 53a$

g) $34z = 66.30$ **h)** $\dfrac{f}{23} = 108.1$ **i)** $\dfrac{j}{1.3} = 1157$

j) $9.4 = \dfrac{h}{9}$ **k)** $6.8n = 8364$ **l)** $136.8 = \dfrac{d}{57}$

3 Work out the value of each letter.

a) $2x + 4 = 10$ $x =$

b) $5a - 10 = 15$ $a =$

c) $18 - 3y = 3$ $y =$

d) $4c + 1 = 9$ $c =$

e) $3z - 5 = 13$ $z =$

f) $5 + 2b = 11$ $b =$

4 Solve these equations. Show all your working.

a) $9d + 18 = 99$

b) $\frac{3}{5}h + 9 = 45$

c) $7d + 3.5 = 70.7$

d) $24 + 5.8 = \frac{e}{4} + 23$

e) $46 - 5s = 4 + 4.2$

f) $5.6 = 3.4 + 9p - 5.9$

g) $4b - 2.4 = 98 - 82$

h) $44 - \frac{d}{2} = 67 - 39$

i) $5.8 + 4.7 = 2.3 + \frac{y}{6}$

Try this

Change these into equations. Solve the equation to find the missing numbers.

a) I think of a number and multiply it by 10. I then subtract 3. The answer is 47.

b) I think of a number and multiply it by 8. I then add 7. The answer is 79.

c) I double a number and then add 22. The answer is 50.

d) I think of a number and divide it by 6. I then subtract 5. The answer is 4.

e) I double a number, add 12 and then subtract 3.5. The answer is 22.5.

Patterns and sequences

You can often find the pattern or rule in a sequence by looking at the difference between the numbers. What is the next number in this sequence?

42   27   12   –3   –18   ☐

Each number is 15 less than the previous one, so the next number is –33. The rule is 'subtract 15'.

A **formula** (plural formulae) uses letters or words to give a rule.
What is the rule for this sequence of numbers?

A	1	2	3	4	5	n
B	2	5	8	11	14	?

Look at the relationship between the pairs of numbers. B = 3A – 1. So for n, the formula is $3n – 1$. You can use this to find any number in the pattern. What is the 15th number? $3 \times 15 – 1 = 44$

1 Write the next two numbers and the rule for each sequence.

a) –16   –11   –6   –1   4   ☐ ☐   the rule is ☐

b) –23   –14   –5   4   13   ☐ ☐   the rule is ☐

c) 438   402   366   330   294   ☐ ☐   the rule is ☐

d) 0.27   0.42   0.57   0.72   0.87   ☐ ☐   the rule is ☐

e) 3.06   1.76   0.46   –0.84   –2.14   ☐ ☐   the rule is ☐

f) 48   47.93   47.86   47.79   47.72   ☐ ☐   the rule is ☐

2 Write the pattern or rule for each sequence. Write **Yes** or **No** for each question.

a) –6   –1   4   9   14   the rule is ☐
Will 99 be in this sequence?

b) –8   –5   –2   1   4   the rule is ☐
Will 30 be in this sequence?

c) 20   14   8   2   –4   the rule is ☐
Will –20 be in this sequence?

d) 11   7   3   –1   –5   the rule is ☐
Will –21 be in this sequence?

e) –52   –22   8   38   68   the rule is ☐
Will 180 be in this sequence?

3 Choose the correct rule for these sequences.

a)

A	1	2	3	4	5	n
B	2	6	10	14	18	?

$2n$ $\qquad$ $3n - 1$ $\qquad$ $4n - 2$ $\qquad$ $2n + 1$

b)

A	1	2	3	4	5	n
B	11	17	23	29	35	?

$5n + 6$ $\qquad$ $6n + 5$ $\qquad$ $4n + 3$ $\qquad$ $8n - 4$

c)

A	1	2	3	4	5	n
B	6	4	2	0	−2	?

$4n - 2$ $\qquad$ $6 - n$ $\qquad$ $4 - n$ $\qquad$ $8 - 2n$

d)

A	1	2	3	4	5	n
B	3	−1	−5	−9	−13	?

$7n - 4$ $\qquad$ $2n - 7$ $\qquad$ $7 - 4n$ $\qquad$ $7 - 2n$

4 For each of the sequences above, what is the value of B if A is 25?

Try this

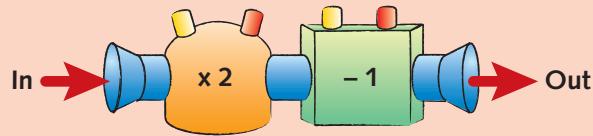| Shape number (S)	1	2	3
Number of rods (R)	5	7	9

The formula for this is R = 2S + 3

a) How many rods will be used for the 6th shape?

b) How many rods will be used for the 20th shape?

c) Which shape number will use 25 rods?

d) What is the formula for this sequence of rods?
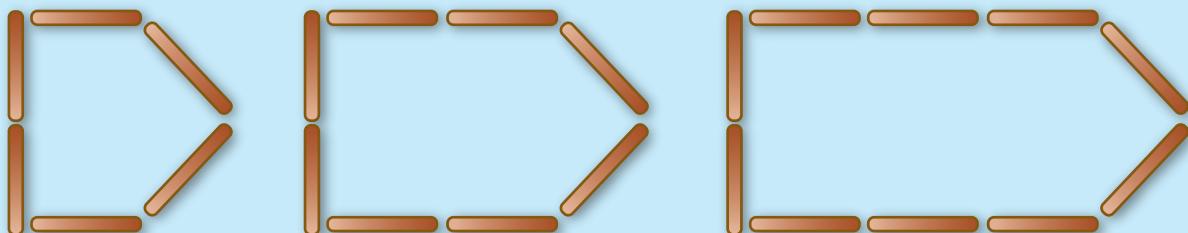
Functions and relationships

The rule for a sequence is: 'multiply by 2 and then subtract 1'.

In ➡ **x 2** **– 1** ➡ Out

The function can be shown as $\triangle = 2\,\square - 1$

This is the table of results showing the sequence.

$\square$	0	1	2	3	4	5
$\triangle$	–1	1	3	5	7	9

1 Copy and complete a table of results for each function.

$\square$	0	1	2	3	4	5
$\triangle$						

a) $\triangle = \square - 6$

b) $\triangle = 2\,\square + 3$

c) $\triangle = 3\,\square - 4$

d) $\triangle = 2\,\square - 10$

e) $\triangle = 4\,\square - 7$

f) $\triangle = 5\,\square + 9$

g) $\triangle = 7 - \square$

h) $\triangle = 5 + 2\,\square$

i) $\triangle = 11 - 6\,\square$

j) $\triangle + 1 = 2\,\square$

k) $8 + \triangle = 3\,\square$

l) $4 + 2\triangle = 3\,\square - 1$

2 Write the function and complete each table of results.

a)

$\square$	0	1	2	3	4	5
$\triangle$	8	9	10	11		

b)

$\square$	0	1	2	3	4	5
$\triangle$	–1	4	9	14		

c)

$\square$	0	1	2	3	4	5
$\triangle$	7	11	15			

d)

$\square$	0	1	2	3	4	5
$\triangle$	6	5	4			

e)

□	0	1	2	3	4	5
△	0	1	2			

f)

□	0	1	2	3	4	5
△	−4	−8	−12			

g)

□	0	1	2	3	4	5
△	7	5	3			

h)

□	0	1	2	3	4	5
△	8	5	2			

Assessment

1 Write the value of each letter in these equations.

a) $7 + 2z = 39$ $\qquad z =$ $\qquad$ **b)** $3g - 6 = 18$ $\qquad g =$

c) $5n - 4 = 56$ $\qquad n =$ $\qquad$ **d)** $79 - 4h = 47$ $\qquad h =$

e) $37 + 4c = 81$ $\qquad c =$ $\qquad$ **f)** $7x + 3 - 8 = 23$ $\qquad x =$

2 Write the rule and the next two numbers in each sequence.

a) the rule is ☐   19   38   57   ☐ ☐

b) the rule is ☐   −9   2   13   ☐ ☐

c) the rule is ☐   125   100   75   ☐ ☐

d) the rule is ☐   32   24   16   ☐ ☐

e) the rule is ☐   −30   −15   0   ☐ ☐

f) the rule is ☐   −1   8   17   ☐ ☐

3 a) What is the function for this sequence? B =

A	0	1	2	3	4	5
B	6	11	16	21	26	31

b) What is the value of B if A is equal to 15?